Radiative Corrections for Lepton Scattering

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Main problem
Accelerated charge radiates

- …While radiative corrections were the largest corrections to the data, and involved a considerable amount of computation, they were understood to a confidence level of 5% to 10% and did not significantly increase the total error in the measurements.

Henry W. Kendall

Nobel Lecture, December 8, 1990

- **Uncertainties in QED radiative corrections limit interpretability of precision experiments on electron-hadron scattering**

Plan of talk

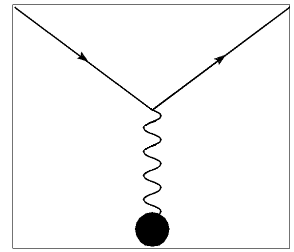
Radiative corrections for electron scattering

- Model-independent and model-dependent; soft and hard photons
- Refined bremsstrahlung calculations
- Two-photon exchange effects in the process *e+p_e+p*

Rad. corrections for electroweak processes

Example:
Measurements of Elastic Nucleon Form Factors

•Based on one-photon exchange approximation



$$M_{fi} = M_{fi}^{1\gamma}$$

$$M_{fi}^{1\gamma} = e^2 \overline{u}_e \gamma_\mu u_e \overline{u}_p (F_1(t)\gamma_\mu - \frac{\sigma_{\mu\nu} q_\nu}{2m} F_2(t)) u_p$$

•Two techniques to measure

$$\sigma = \sigma_0 (G_M{}^2 \tau + \varepsilon \cdot G_E{}^2) \quad : Rosenbluth \quad technique$$

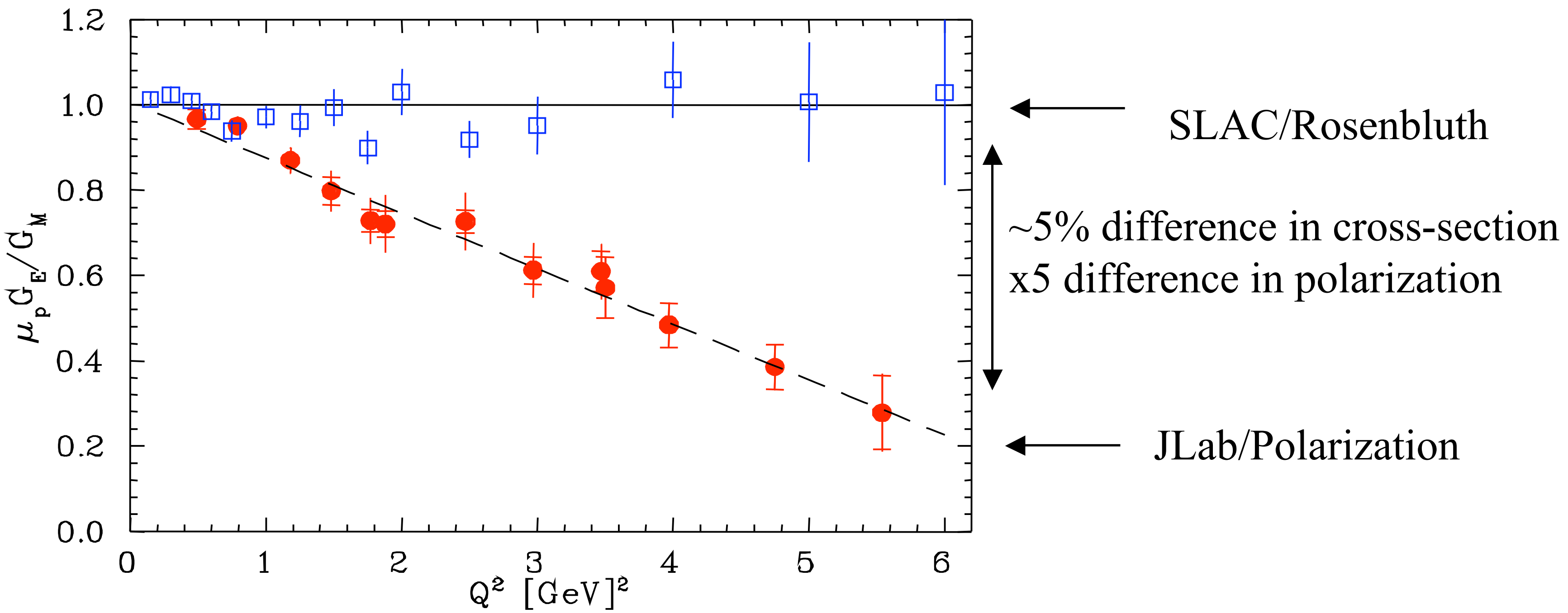
$$\frac{P_x}{P_z} = -\frac{A_x}{A_z} = -\frac{G_E \sqrt{\tau}\sqrt{2\varepsilon(1-\varepsilon)}}{G_M \tau \sqrt{1-\varepsilon^2}} \quad : Polarization \quad technique$$

$$G_E = F_1 - \tau F_2, \quad G_M = F_1 + F_2$$

$$(P_y = 0)$$

Latter due to: Akhiezer, Rekalo; Arnold, Carlson, Gross

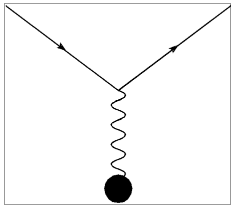
Do the techniques agree?



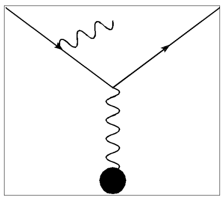
- **Both early SLAC and Recent JLab experiments on (super)Rosenbluth separations followed Ge/Gm~const**
- **JLab measurements using polarization transfer technique give different results (Jones'00, Gayou'02)**

Radiative corrections, in particular, a short-range part of 2-photon exchange is a likely origin of the discrepancy

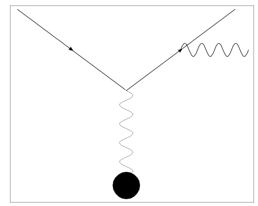
Basics of QED radiative corrections



(First) Born approximation

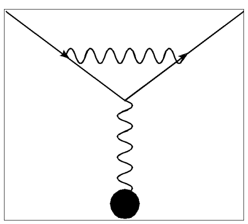


Initial-state radiation



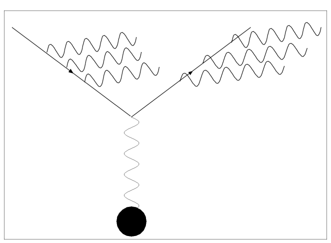
Final-state radiation

Cross section ~ d_/_ => integral diverges logarithmically: **IR catastrophe**



Vertex correction => cancels divergent terms; Schwinger (1949)

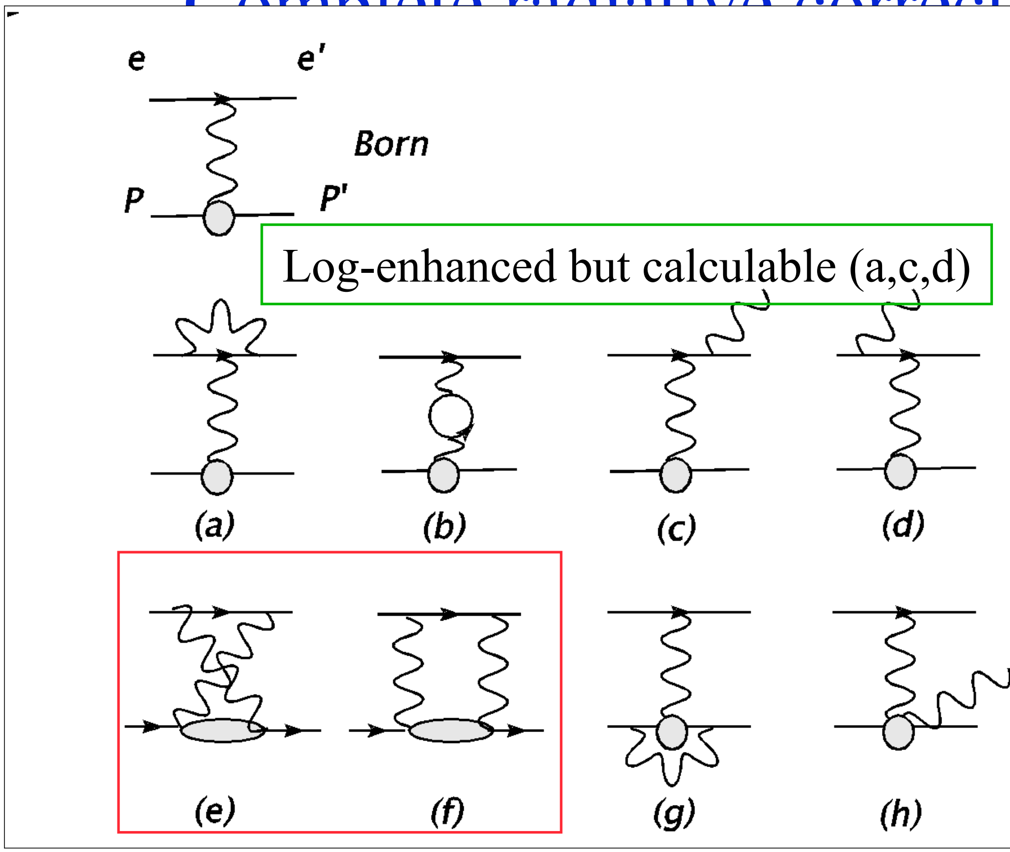
$$\sigma_{\exp} = (1+\delta)\sigma_{Born}, \quad \delta = \frac{-2\alpha}{\pi}\{(\ln\frac{E}{\Delta E} - \frac{13}{12})(\ln\frac{Q^2}{m_e^2} - 1) + \frac{17}{36} + \frac{1}{2}f(\theta)\}$$



Multiple soft-photon emission: solved by exponentiation, Yennie-Frautschi-Suura (YFS), 1961

$$(1+\delta) \to e^{\delta}$$

Complete radiative correction in O(α_{em})



Radiative Corrections:

- Electron vertex correction (a)
- Vacuum polarization (b)
- Electron bremsstrahlung (c,d)
- Two-photon exchange (e,f)
- Proton vertex and VCS (g,h)
- Corrections (e-h) depend on the nucleon structure

•Meister&Yennie; Mo&Tsai

•Further work by Bardin&Shumeiko; Maximon&Tjon; AA, Akushevich, Merenkov;

•Guichon&Vanderhaeghen'03:
Can (e-f) account for the Rosenbluth vs. polarization experimental discrepancy? Look for ~3% ...

Main issue: Corrections dependent on nucleon structure

Model calculations:

•Blunden, Melnitchouk,Tjon, Phys.Rev.Lett.**91**:142304,2003

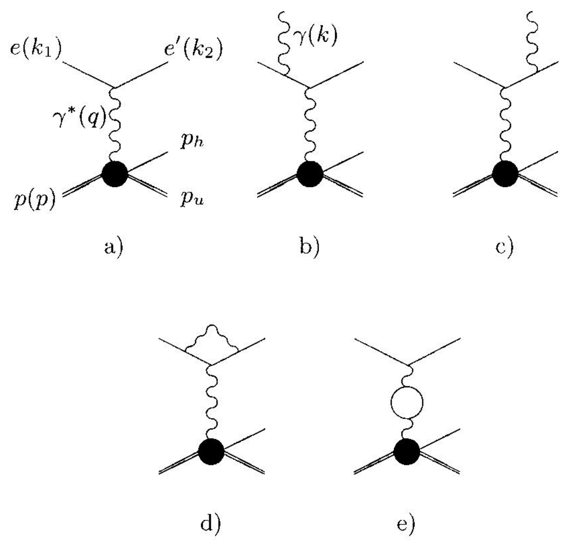
•Chen, AA, Brodsky, Carlson, Vanderhaeghen, Phys.Rev.Lett.**93**:122301,2004

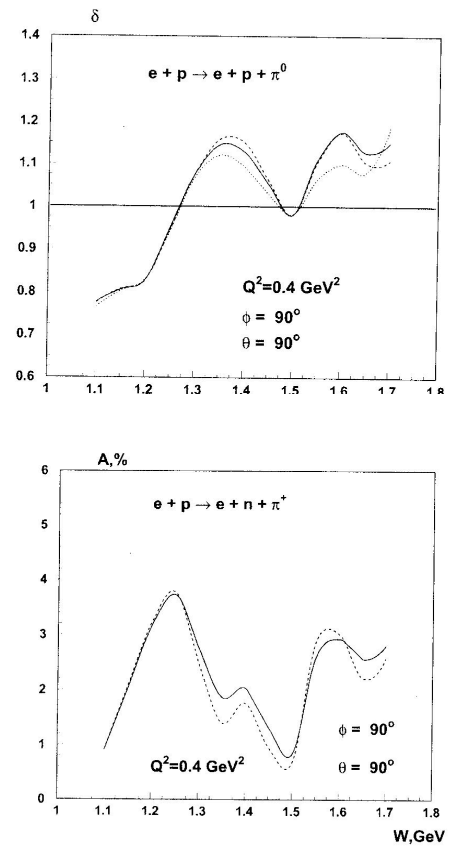
Basic Approaches to QED Corrections

- L.W. Mo, Y.S. Tsai, Rev. Mod. Phys. 41, 205 (1969); Y.S. Tsai, Preprint SLAC-PUB-848 (1971).
  - Considered both elastic and inelastic inclusive cases. No polarization.
- D.Yu. Bardin, N.M. Shumeiko, Nucl. Phys. B127, 242 (1977).
  - Covariant approach to the IR problem. Later extended to inclusive, semi-exclusive and exclusive reactions with polarization.
- E.A. Kuraev, V.S. Fadin, Yad.Fiz. 41, 7333 (1985); E.A. Kuraev, N.P.Merenkov, V.S. Fadin, Yad. Fiz. 47, 1593 (1988).
  - Developed a method of electron structure functions based on Drell-Yan representation; currently widely used at e^+e^- colliders.

RC for Electroproduction of Pions

- **AA, Akushevich, Burkert, Joo, Phys.Rev.D66, 074004 (2002)**
  - Conventional RC, precise treatment of phase space, no peaking approximation, no dependence on hard/soft photon separation
  - Can be used for any exclusive electroproduction of 2 hadrons, e.g., d(e,e'p)n (EXCLURAD code)





See http://www.jlab.org/RC for other codes
Used in data analysis at JLab
(and MIT, HERMES, MAMI,…)

Bethe-Heitler corrections to polarization transfer and cross sections

AA, Akushevich, Merenkov Phys.Rev.D64:113009,2001;
AA, Akushevich, Ilychev, Merenkov, PL B514, 269 (2001)

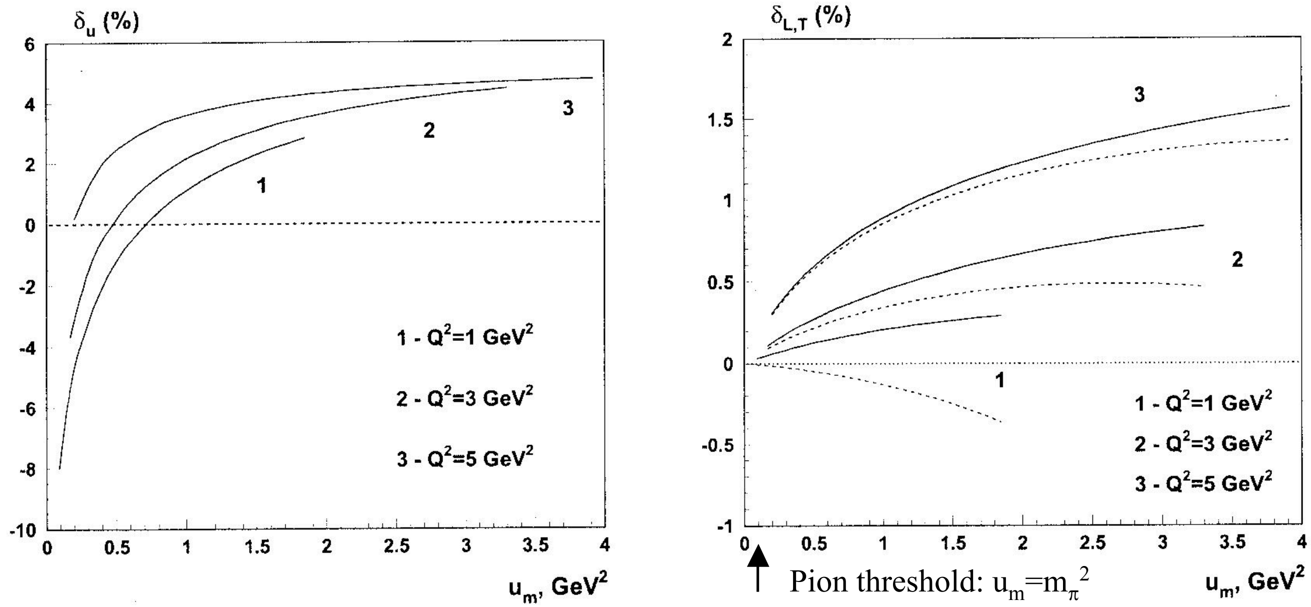
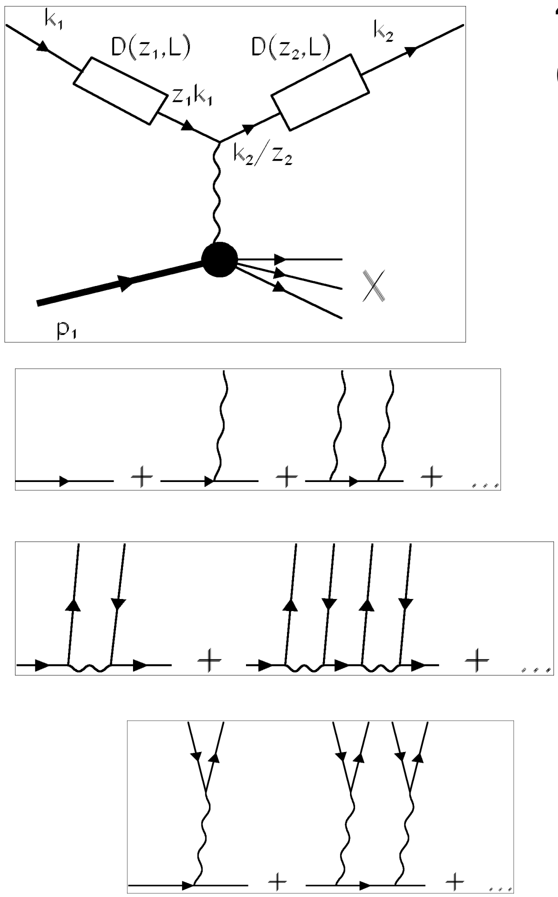


Figure 2: Radiative corrections to the unpolarized cross section (left plot) and polarization asymmetries (right plot) defined in (41). Solid and dashed lines corresponds to longitudinal and transverse cases. S=8 GeV^2.

In kinematics of elastic ep-scattering measurements, cross sections are more sensitive to RC

Electron Structure Functions (Kuraev,Fadin,Merenkov)

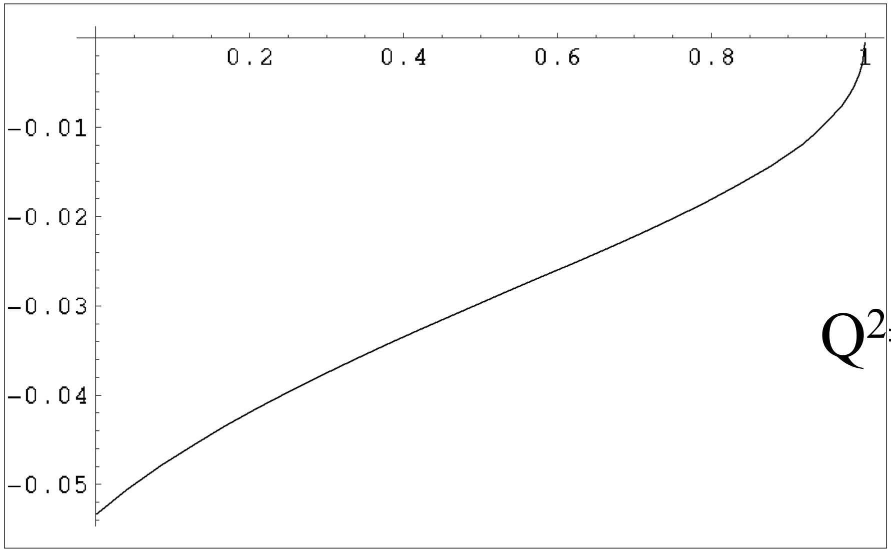


- For polarized ep->e'X scattering, AA et al, JETP 98, 403 (2004); elastic ep: AA et al. PRD 64, 113009 (2001).
  - Resummation technique for collinear photons (=peaking approx.)
  - Difference <0.5% from previous calculation including hard brem

Separating *soft* 2-photon exchange

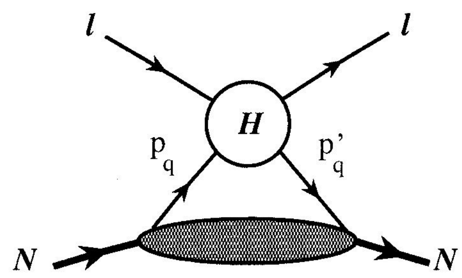
- Tsai; Maximon & Tjon (k_0)
- Grammer &Yennie prescription PRD 8, 4332 (1973) (also applied in QCD calculations)
- Shown is the resulting (soft) QED correction to cross section
- **Already included in experimental data analysis**
- NB: Corresponding effect to polarization transfer and/or asymmetry is zero

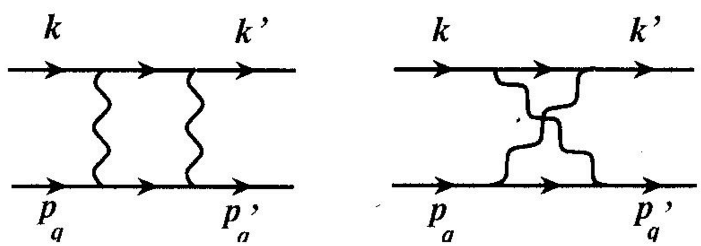
Soft



$Q^2 = 6\ \text{GeV}^2$

Calculations using Generalized Parton Distributions





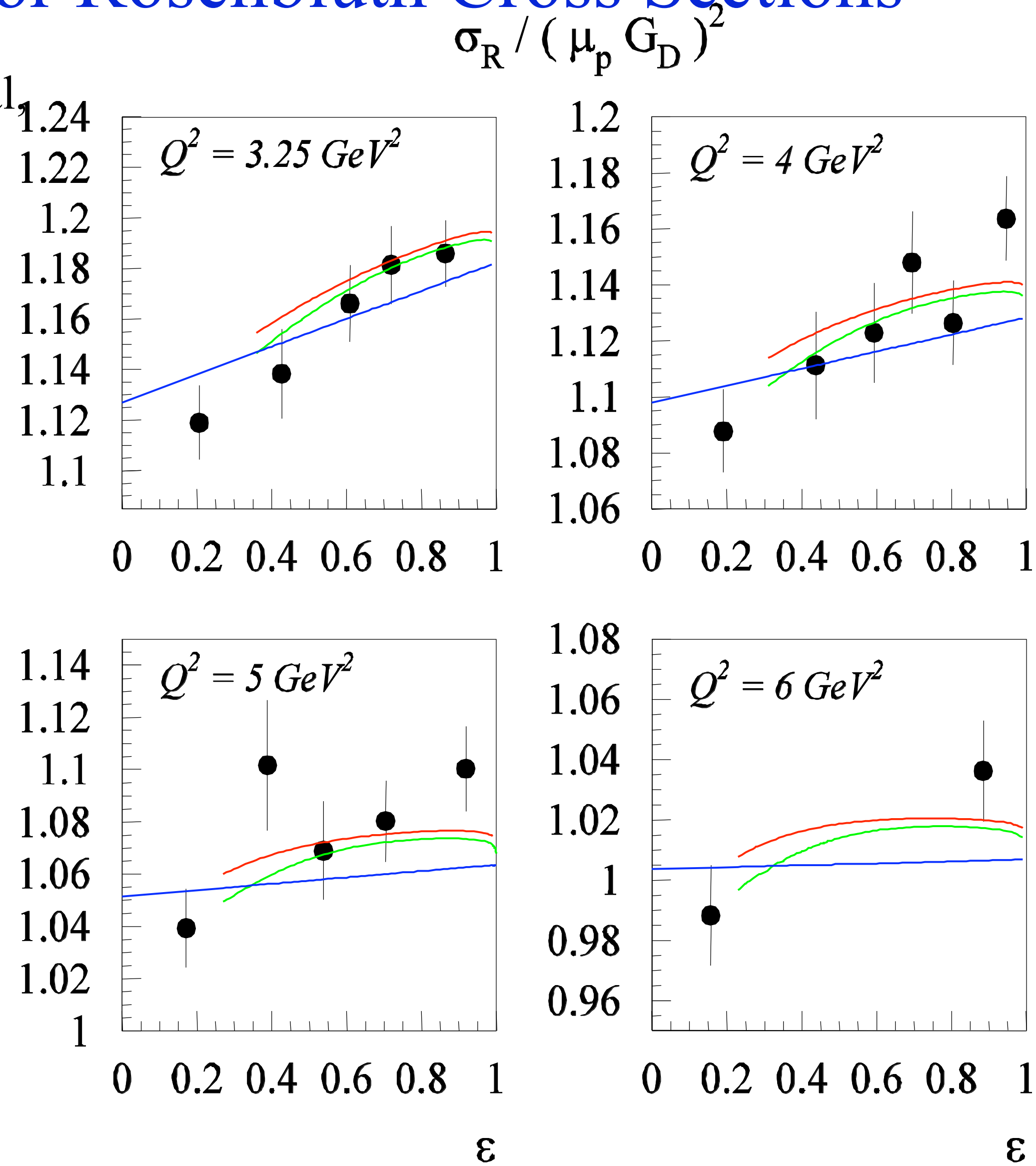
Model schematics:
- Hard eq-interaction
- GPDs describe quark emission/absorption
- Soft/hard separation
  - Use Grammer-Yennie prescription

Hard interaction with a quark

AA, Brodsky, Carlson, Chen, Vanderhaeghen, Phys.Rev.Lett.**93**:122301,2004; Phys.Rev.D**72**:013008,2005

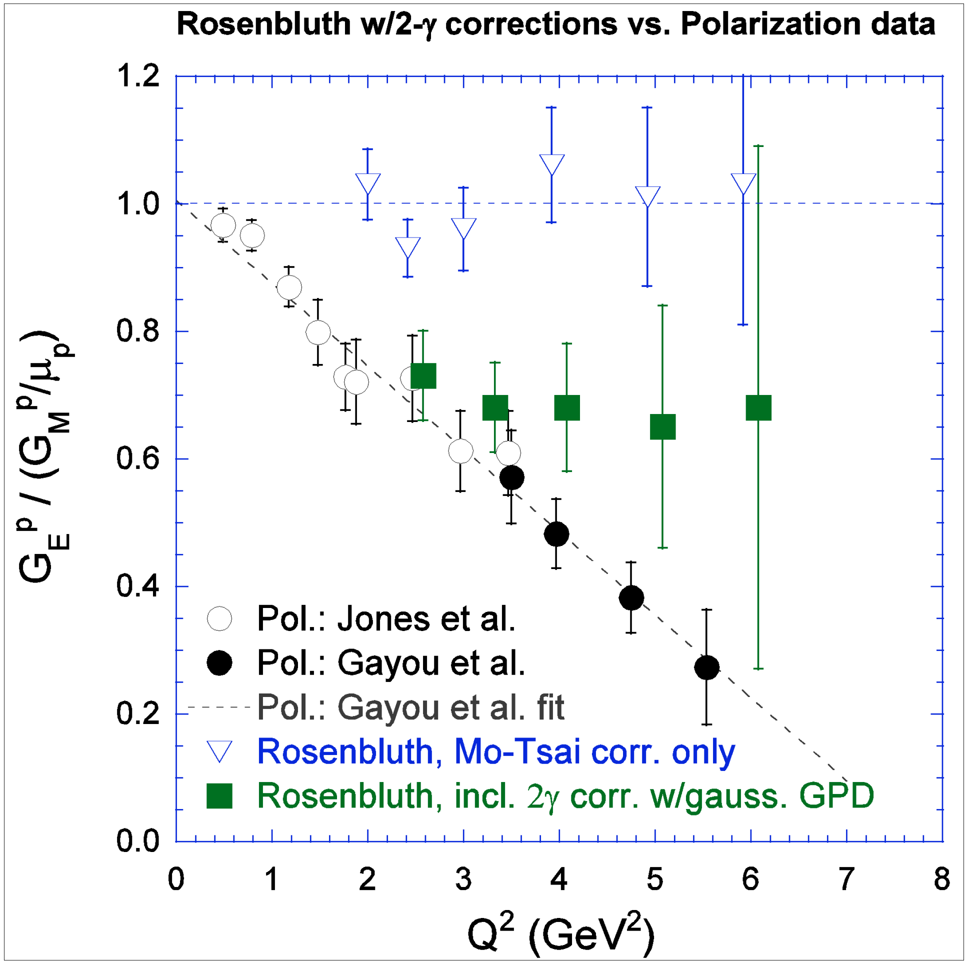
Two-Photon Effect for Rosenbluth Cross Sections

- Data shown are from Andivahis et al, PRD 50, 5491 (1994)
- Included GPD calculation of two-photon-exchange effect
- Qualitative agreement with data:
- Discrepancy likely reconciled



Updated Ge/Gm plot

AA, Brodsky, Carlson, Chen, Vanderhaeghen,
Phys.Rev.Lett.93:122301, 2004; Phys.Rev.D72:013008, 2005



Full Calculation of Bethe-Heitler Contribution

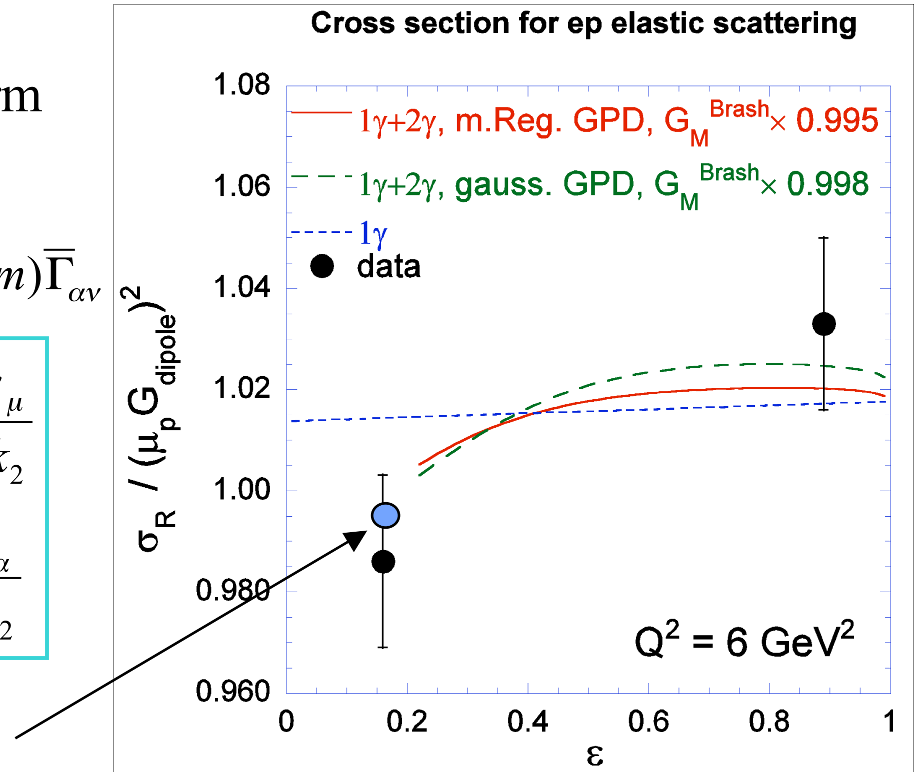
*Additional work by AA et al., using MASCARAD (**Phys.Rev.D64:113009,2001**)*
Full calculation including soft and hard bremsstrahlung

Radiative leptonic tensor in full form
AA et al, ***PLB 514, 269 (2001)***

$$L^{r}_{\mu\nu} = -\frac{1}{2}Tr(\hat{k}_2 + m)\Gamma_{\mu\alpha}(1+\gamma_5\hat{\xi}_e)(\hat{k}_1 + m)\overline{\Gamma}_{\alpha\nu}$$

$$\Gamma_{\mu\alpha} = \left(\frac{k_{1\alpha}}{k\cdot k_1} - \frac{k_{2\alpha}}{k\cdot k_2}\right)\gamma_\mu - \boxed{\frac{\gamma_\mu \hat{k}\gamma_\alpha}{2k\cdot k_1} - \frac{\gamma_\alpha \hat{k}\gamma_\mu}{2k\cdot k_2}}$$

$$\Gamma_{\alpha\nu} = \left(\frac{k_{1\alpha}}{k\cdot k_1} - \frac{k_{2\alpha}}{k\cdot k_2}\right)\gamma_\nu - \boxed{\frac{\gamma_\alpha \hat{k}\gamma_\nu}{2k\cdot k_1} - \frac{\gamma_\nu \hat{k}\gamma_\alpha}{2k\cdot k_2}}$$

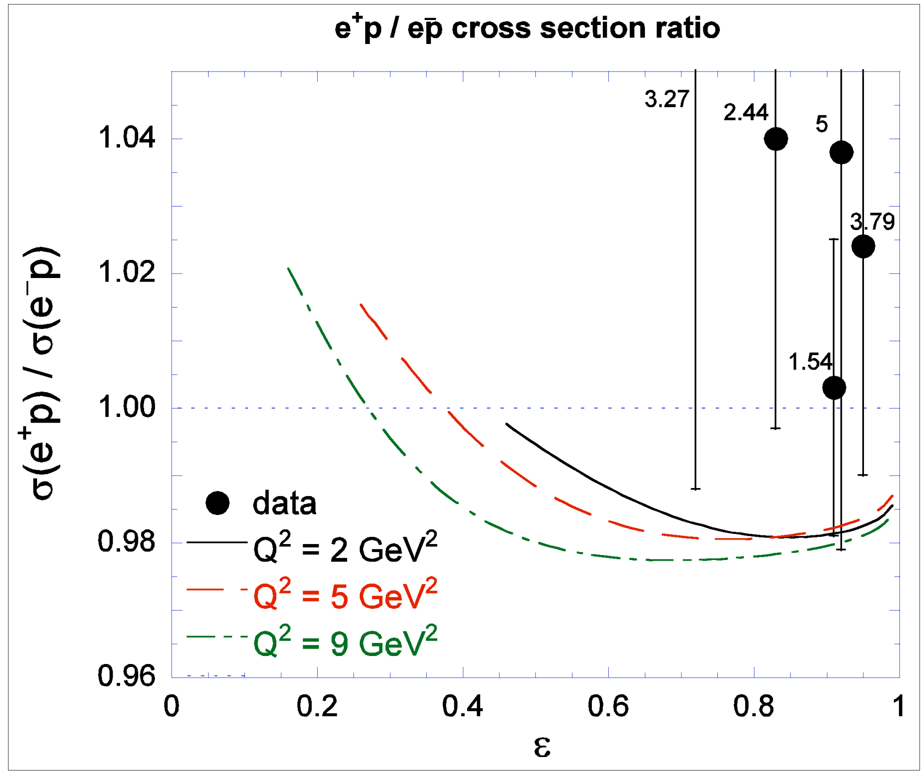


Additional effect of full soft+hard brem _ +1.2% correction to _-slope

Resolves additional ~25% of Rosenbluth/polarization discrepancy!

Charge asymmetry

- Cross sections of electron-proton scattering and positron-proton scattering are equal in one-photon exchange approximation
  - Different for two- or more photon exchange



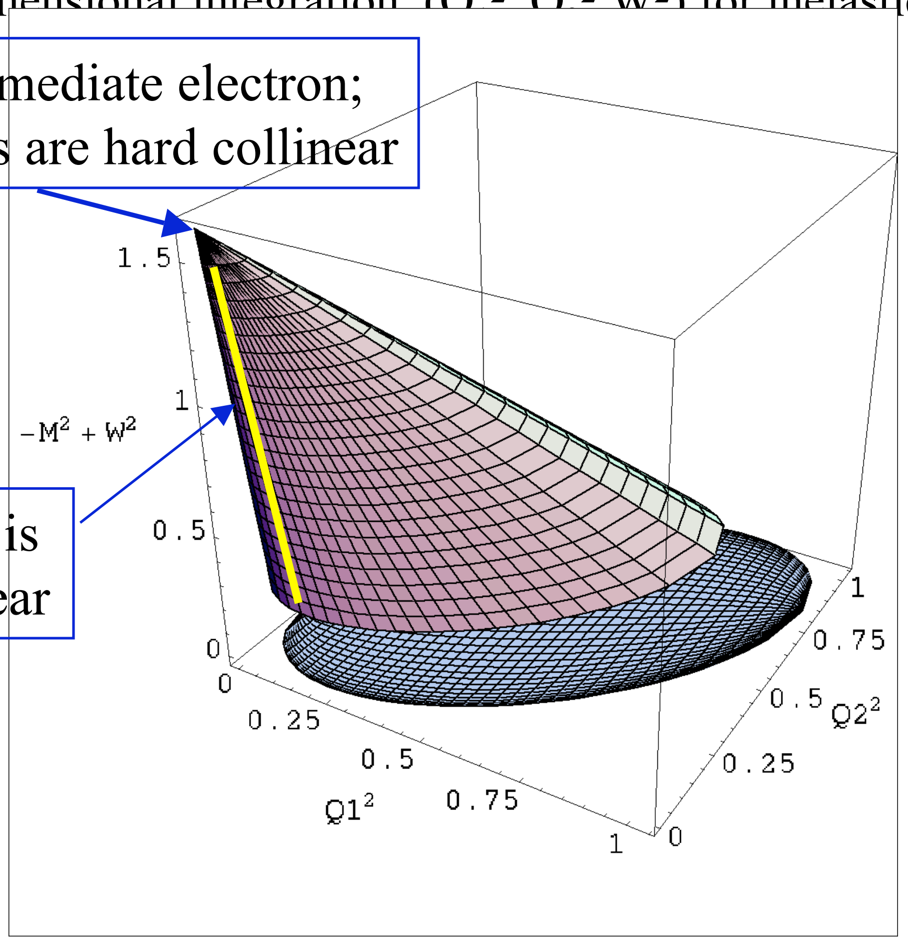
To be measured in JLab Experiment 04-116, Spokepersons AA, W. Brooks, L.Weinstein, et al.

Phase Space Contributing to the absorptive part of 2_-exchange amplitude

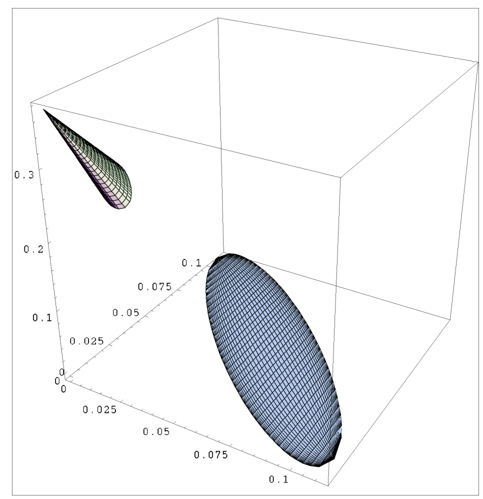
- 2-dimensional integration (Q_1^2, Q_2^2) for the elastic intermediate state
- 3-dimensional integration (Q_1^2, Q_2^2, W^2) for inelastic excitations

`Soft' intermediate electron;
Both photons are hard collinear

One photon is
Hard collinear



Examples: MAMI A4
E= 855 MeV
_cm= 57 deg;
SAMPLE, E=200 MeV



Other theoretical developments

- Blunden et al., Phys.Rev.C72:034612, 2005
  Approximate proton Compton amplitude by Born terms
- Kondratyuk et al., nucl-th/0506026
  Add intermediate _-excitation to the above
- Pascalutsa et al., hep-ph/0509055
  GPD approach extended to N__ transition
- Borisyuk, Kobushkin, Phys.Rev.C72:035207,2005

Future task: Resummation of inelastic excitations at lower Q^2

Two-photon exchange for electron-proton scattering

- Quark-level short-range contributions are substantial (3-4%) ; correspond to J=0 fixed pole (Brodsky-Close-Gunion, PRD 5, 1384 (1972)).
- Structure-dependent radiative corrections calculated using GPDs bring into agreement the results of polarization transfer and Rosenbluth techniques for Gep measurements
- Experimental tests of two-photon exchange
  - Comparison between electron and positron elastic scattering (JLab E04-116)
  - Measurement of nonlinearity of Rosenbluth plot (JLab E05-017)
  - Search for deviation of angular dependence of polarization and/or asymmetries from Born behavior at fixed Q^2 (JLab E04-019)
  - Elastic single-spin asymmetry or induced polarization (JLab E05-015)
  - 2_ additions for parity-violating measurements (HAPPEX, G0)

Through active theoretical support emerged a research program of

Testing precision of the electromagnetic probe

Double-virtual VCS studies with two space-like photons

Radiative Corrections for Electro-Weak Processes

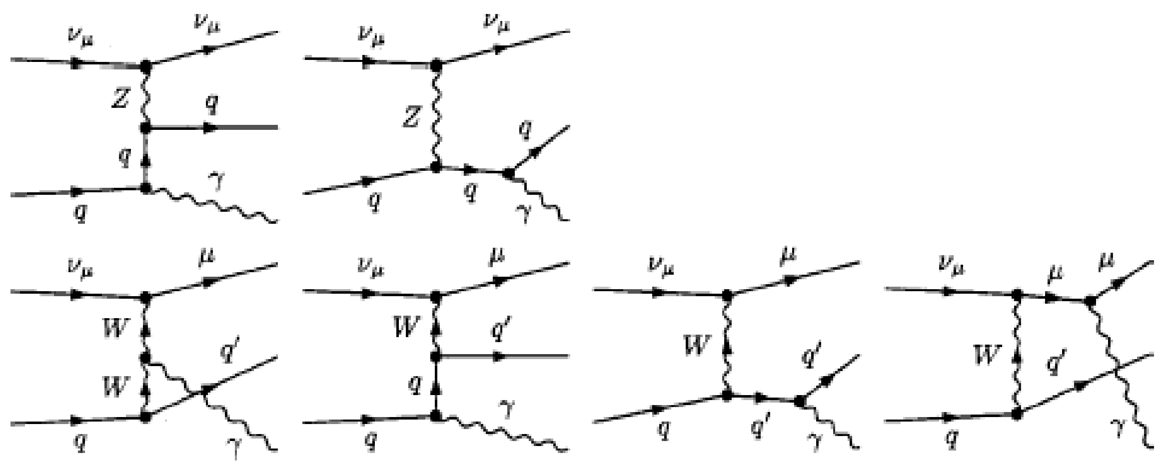
Semi-Leptonic processes involving nucleons

- Neutrino-nucleon scattering

  Per cent level reached by NuTeV. Radiative corrections for DIS calculated at a partonic level (D. Bardin et al.)

- Neutron beta-decay: Important for V_{ud} measurements; axial-vector coupling g_A Marciano, Sirlin, PRL 56, 22 (1986); Ando et al., Phys.Lett.B595:250-259,2004; Hardy, Towner, PRL94:092502,2005
  - Extended to _N by Fukugita, Acta Phys.Polon.B35:1687-1732,2004; and _D: Phys.Rev.D72:071301,2005, Erratum-ibid.D74:039906,2006;
  - Kurylov, Phys.Rev. C65 (2002) 055501=> ~4% effect for _$_{total}$
- Parity-violating DIS: Bardin, Fedorenko, Shumeiko, Sov.J.Nucl.Phys.32:403,1980; J.Phys.G7:1331,1981, up to 10% effect from rad.corrections
- Parity-violating elastic ep (strange quark effects, weak mixing angle)

Implications for Nutev



- Diener, Dittmaier, Hollik, Phys.Rev.D69:073005,2004:
  - **Rad.Corrections used by NuTeV likely underestimated**,

  "*…we compare results that differ in the input-parameter scheme, treatment of real photon radiation, and factorization scheme. The associated shifts in the theoretical prediction for the ratio of neutral- and charged-current cross sections can be larger than the experimental accuracy of the NuTeV result. ...*"

Neutrino DIS

- Arbuzov, Bardin, Kalinovskaya, JHEP06(2005)078

_uNC(%)

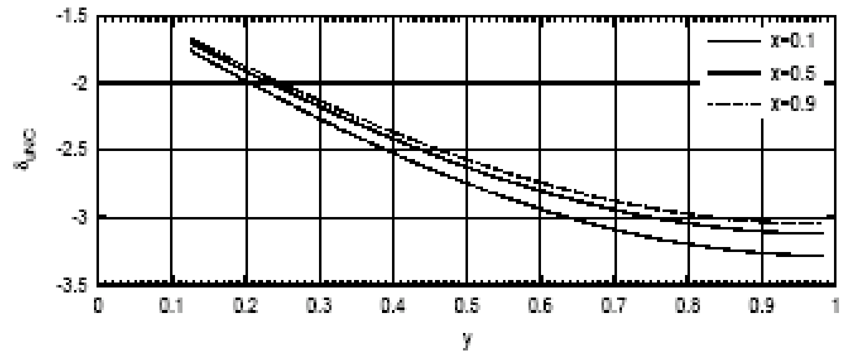


Figure 1: Relative effect of radiative corrections to $\nu - u$ NC scattering as a function of y for three fixed values of x.

E_=80GeV

_dCC(%)

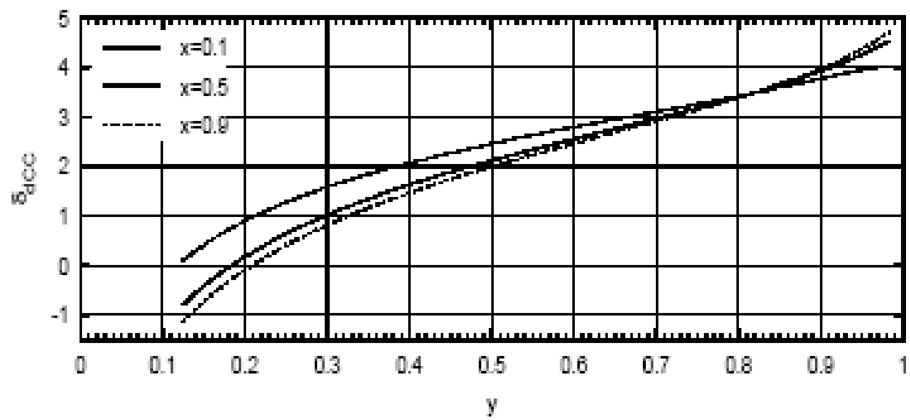
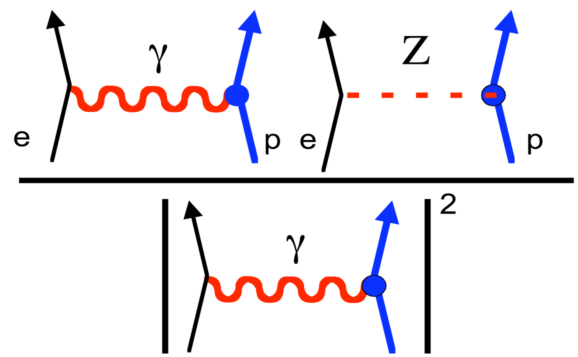


Figure 2: Relative effect of radiative corrections to $\nu - d$ CC scattering as a function of y for three fixed values of x.

Parity Violating elastic e-N scattering

Longitudinally polarized electrons,
unpolarized target

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \left[\frac{-G_F Q^2}{4\pi\alpha\sqrt{2}}\right] \frac{A_E + A_M + A_A}{2\sigma_{unpol}}$$



$$\boxed{\begin{aligned} A_E &= \varepsilon(\theta) G_E^Z G_E^\gamma \\ A_M &= \tau G_M^Z G_M^\gamma \\ A_A &= -(1-4\sin^2\theta_W)\varepsilon' G_A^e G_M^\gamma \end{aligned}}$$

$\tau = Q^2/4M^2$
$\varepsilon = [1+2(1+\tau)\tan^2(\theta/2)]^{-1}$
$\varepsilon' = [\tau(\tau+1)(1-\varepsilon^2)]^{1/2}$

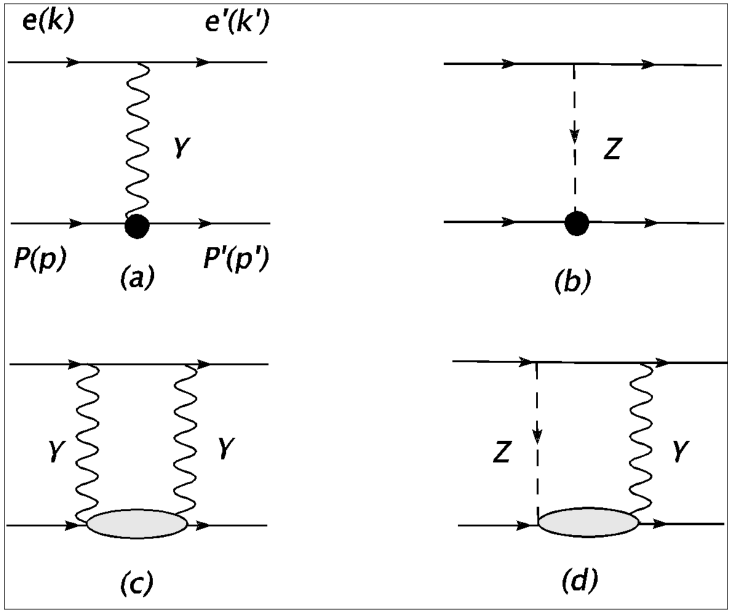
Neutral weak form factors contain explicit contributions from strange sea

$$\boxed{\begin{aligned} G_{E,M}^Z(Q^2) &= (1-4\sin^2\theta_W)(1+R_A^p)G_{E,M}^p - (1+R_A^n)G_{E,M}^n - G_{E,M}^s \\ G_A^e(Q^2) &= -G_A^Z + (\eta F_A^\gamma + R^e) + \Delta s \end{aligned}}$$

$$\eta = \frac{8\pi\alpha\sqrt{2}}{1-4\sin^2\theta_W} = 3.45$$

$G^Z{}_A(0) = 1.2673 \pm 0.0035$ (from β decay)

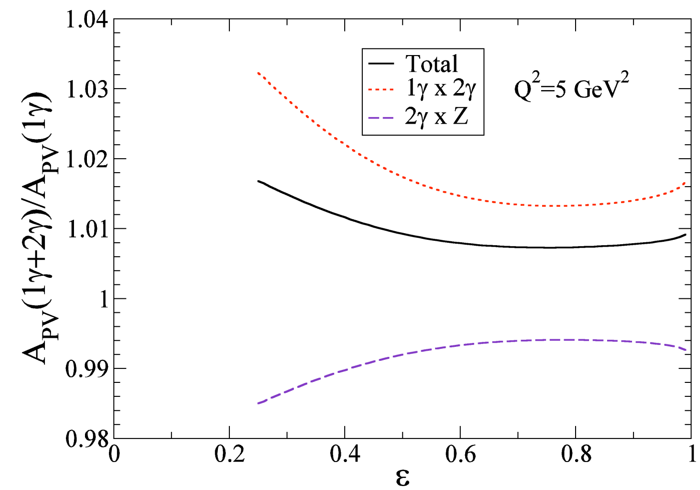
Born and Box diagrams for elastic ep-scattering

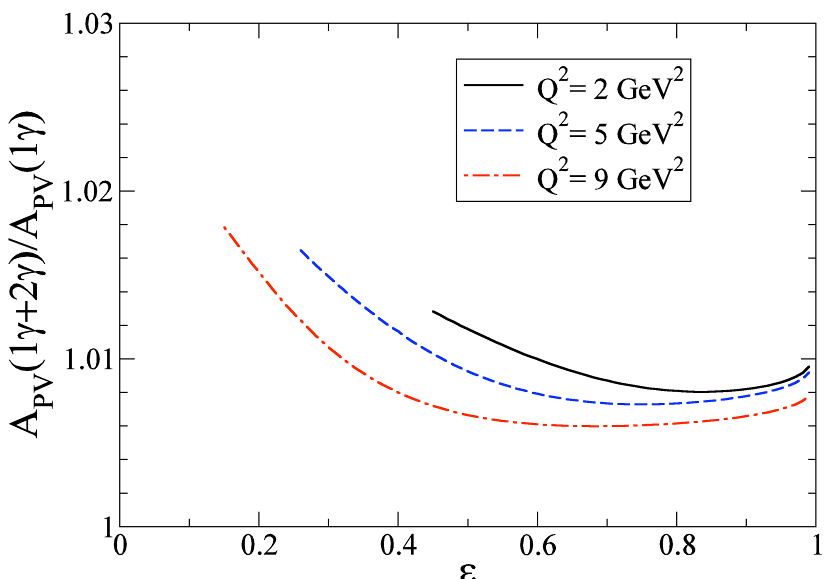


- (d) Computed by Marciano,Sirlin, Phys.Rev.D29:75,1984, Erratum-ibid.D31:213,1985 for atomic PV (i.e., Q^2 _0)
- (c) Presumed small, e.g., M. Ramsey-Musolf, Phys.Rev. C60 (1999) 015501

GPD Calculation of 2__Z-interference

- Can be used at higher Q^2, but points at a problem of additional systematic corrections for parity-violating electron scattering. The effect evaluated in GPD formalism is the largest for backward angles:





AA & Carlson, ***Phys. Rev. Lett. 94, 212301 (2005):***
Measurements of strange-quark content of the nucleon are affected, _s may shift by ~10%

Important note: (nonsoft) 2_-exchange amplitude has no $1/Q^2$ singularity; 1_-exchange is $1/Q^2$ singular => At low Q2, 2_-corrections is suppressed as Q^2 P. Blunden used this formalism and evaluated correction of 0.16% for Q_{weak}

2_-correction for ep-scattering via Z-exchange

- 2_-correction to parity-violating asymmetry **does not cancel**. May reach a few per cent for GeV momentum transfers
- Corrections are angular-dependent, not reducible to re-definition of coupling constants
  - Revision of _Z-box contribution and extension of model calculations to lower Q^2 is necessary
- Further developments:
  - Zhu, Kao, Yang, Phys.Rev.Lett.99:262001,2007: Found essential Q2-dependence of EW box contributions
  - Tjon, Melnitchouk, arXiv:0711.0143 [nucl-th]: Model calculation of EW box

RC for Minerva

- For CC cross sections, anticipate 1-5% electromagnetic effects
  - Bremsstrahlung calculations model-independent, but need to be matched with experimental cuts and acceptances
  - Electroweak box diagrams calculations depend on the used model of hadronic structure; can be constrained by existing (and forthcoming) info on 2_-exchange for elastic ep-scattering

- *Expertise at JLab available to implement Rad.Corrections for data analysis of Minerva*